Exam 2015 Solution

1. The swap curve

1.1 Swap curve intuition

1.1.1

The par swap rate is a weighted average of the forward rates. We discount and project forward rates on the same curve, which is flat at 2%. Hence the par swap rate will be roughly 2% for all maturities. Actually it will be slightly above 2%, since the fixed leg day count convention is 30/360, whereas the floating leg convention is Act/360 (see next question).

This makes intuitive sense. Since the floating leg always pays 2% on this curve, the fixed rate must be close to the same for the IRS to be fair (have NPV 0).

1.1.2

Par swap rates will decrease. We now use Act/360 instead of 30/360, which on average will give larger coverages (there are 365 days in a year, not 360). To compensate for this, the swap rates must decrease to maintain a NPV of 0.

1.1.3

A payer swap is positioned for rates to go up. Hence you will make money (positive profit).

Since we are paying a fixed rate but receiving a floating (variable) rate, we receive more on the higher curve, but pay the same. That gives positive profit.

Increasing rates has two offsetting effects. The direct effect is that we project higher forward rates. The secondary effect is that we will discount future cash flows harder (make them worth less). For small rate changes the direct effect dominates an ATM trade, and the profit relationship is linear. For greater changes the trade will gain considerable present value, and the discounting effect becomes significant, introducing non-linearity.

1.1.4

The risk reveals a 5Y5Y payer IRS which is out-of-the-money (negative NPV).

A 5Y5Y forward starting payer swap can be thought of as a 10Y payer swap and a 5Y receiver swap. Such a combination would give rise to considerable negative forward curve risk in the 5Y bucket, and considerable positive forward curve risk in the 10Y bucket.

A trade which has a negative NPV will gain when we increase the discounting curve, since we discount the future liability harder.

1.1.5

Our hedge would be a 5Y5Y receiver swap with a larger notional than the original.

If we use the same notional as on the original trade, we would eliminate all risk on the forward curve, but be left with outright risk (stemming from the discounting curve). We have to do the trade ATM, so we cannot get rid of the discounting curve risk. However, we can compensate for it by increasing the trade notional, which will increase the forward curve risk, and leave us with no outright exposure.

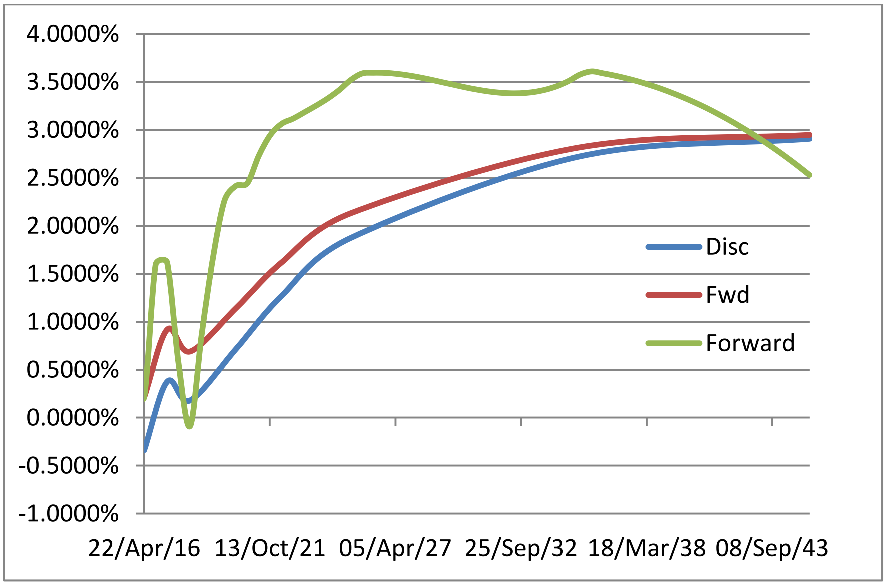
1.2 Detecting a calibration error

1.2.1 and 1.2.2

Using the solver, allowing negative rates yields.

Knot points	Disc	Fwd
22-apr-16	-0.3386%	0.2131%
24-apr-17	0.3756%	0.9197%
23-apr-18	0.1796%	0.6910%
22-apr-20	0.7157%	1.1423%
22-apr-22	1.2713%	1.6202%
22-apr-25	1.8726%	2.1304%
23-apr-35	2.7226%	2.8173%
24-apr-45	2.9059%	2.9467%

1.2.3



1.2.4

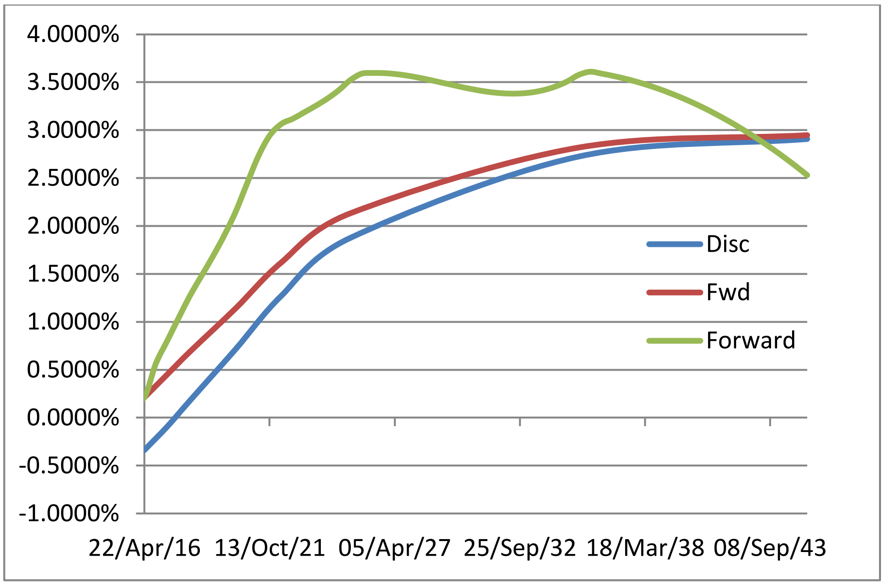
Inspecting your curves reveals that the problem occurs in the 2Y to 3Y region. By looking at the data, we see the 2Y IRS rate is higher than the 3Y rate, which on this increasing swap curve makes no sense. Either the 2Y rate is too high, or the 3Y rate is too low. Trials and error reveals that changing the 2Y point gives the most smooth forward curve.

The correct 2Y par swap rate is 0.4548%.

1.2.5

Using the cleaned data yields.

Knot points	Disc	Fwd
22-apr-16	-0.3386%	0.2131%
24-apr-17	-0.0918%	0.4529%
23-apr-18	0.1800%	0.6915%
22-apr-20	0.7166%	1.1430%
22-apr-22	1.2725%	1.6212%
22-apr-25	1.8739%	2.1314%
23-apr-35	2.7236%	2.8181%
24-apr-45	2.9067%	2.9473%



1.3 Evaluating and hedging a swap portfolio

1.3.1

The present value of the portfolio of swaps is 117,944,620. We accept a certain amount of slack here (plus or minus 2 million), since slight calibration differences will result in a different PV.

1.3.2

Model rate delta vector			
	Portfolio		
Knot points	Disc	Fwd	Net
22-apr-16	-662	44,322	43,660
24-apr-17	-944	-46,006	-46,950
23-apr-18	-7,647	34,884	27,237
22-apr-20	-11,599	331,862	320,263
22-apr-22	10,294	-4,554	5,740
22-apr-25	-44,730	-415,983	-460,714
23-apr-35	-59,382	253,498	194,116
24-apr-45	-2,266	88,002	85,736
Total	-116,937	286,025	169,088

Again, we accept slight differences in the risk numbers.

We are outright positioned for higher rates. That is, if both the forward- and discounting curve is moved upwards in a parallel shift, we will earn EUR 169,088.
We see a 5Y10Y flattener and a 10Y20Y steepener. Hence we are positioned for less curvature.
We are positioned for a greater spread between discounting- and forward rates, since such a move would either imply a higher forward curve, or lower discounting curve.

1.3.3

The Jacobian trick yields.

Market rate delta vector			
		Portfolio	
Start	Maturity	IRS	CCS
2b	1Y	43,625	-821
2b	2Y	-46,554	-899
2B	3Y	26,822	-4,882
2B	5Y	327,738	-9,148
2B	7Y	30,780	-1,894
2B	10Y	-547,722	-32,856
2B	20Y	204,964	-59,226
2B	30Y	130,452	-5,393
	Total	170,106	-115,118

1.3.4

Inspecting the market rate delta vector, we see that the main exposure is towards 5Y, 10Y, 20Y, 30Y IRSs, and 10Y, 20Y CCSs. How you hedge this is part art, part science. It depends on liquidity and other market conditions for the available hedges that you need to know as a trader.

We would use a 5Y receiver IRS to hedge all IRS risk out to 5Y. Then a 10Y payer IRS to hedge the 7Y and 10Y IRS risk, and a 20Y IRS to hedge the 20Y and beyond IRS risk. To take care of the CCS risk, we would bucket everything in a 20Y payer CCS.

	Correct Notional
Hedge IRS 5Y	710,823,456
Hedge IRS 10Y	551,339,363
Hedge IRS 20Y	206,731,098
Hedge CCS 20Y	69,301,492

The notionals can vary quite substantially depending on your hedge. Applying the hedges yields the following net risk.

Model rate delta vector			
	Hedged Portfolio		
Knot points	**Disc**	**Fwd**	**Net**
22-apr-16	-822	44,684	43,862
24-apr-17	-986	-45,914	-46,901
23-apr-18	-5,053	31,581	26,529
22-apr-20	-9,182	-14,665	-23,847
22-apr-22	-4,020	32,839	28,820
22-apr-25	-18,287	-15,662	-33,949
23-apr-35	42,479	-125,294	-82,815
24-apr-45	-4,016	92,352	88,336
Total	113	-79	34
Market rate delta vector			
		Hedged Portfolio	
Start	**Maturity**	**IRS**	**CCS**
2b	**1Y**	43,625	-821
2b	**2Y**	-46,554	-899
2B	**3Y**	26,822	-4,881
2B	**5Y**	-23,899	-9,148
2B	**7Y**	30,782	-1,891
2B	**10Y**	-30,734	-32,867
2B	**20Y**	-130,372	55,890
2B	**30Y**	130,452	-5,394
	Total	122	-11

The important thing here is that both model- and market rate delta vector should be significantly reduced. There will always be some residual risk in such exercises.

1.4 Building a VBA function

This can generally be solved in 2 ways. Either by making one big function that combines FidAnnuityPv, FidFloatingPv, and FidSwapPv. The alternative is to change the 3 functions separately. Either way works. The important thing is to realize that you should make some sort of loop over notionals.

I have highlighted my changes to the already available functions by underlining them.

```
Public Function fidAnnuityPvAmort(AnchorDate As Date, StartDate As Variant, Maturity As Variant, Tenor As String, DayCountBasis As String, DayRule As String, Notional As Double, DiscCurveMat As Variant, DiscCurveRates As Variant, Method As String) As Double

Dim temp As Variant, PayDate As Date, Cvg As Double
Dim i As Integer, n As Integer, noti As Double

temp = fidGenerateSchedule(AnchorDate, StartDate, Maturity, Tenor, DayCountBasis, DayRule)

n = UBound(temp)
For i = 1 To n
PayDate = temp(i, 4)
Cvg = temp(i, 5)
noti = (n - i + 1) / n * Notional
fidAnnuityPvAmort = fidAnnuityPvAmort + Cvg * fidDiscFactor(AnchorDate, PayDate, DiscCurveMat, DiscCurveRates, Method) * noti
Next i

End Function

Public Function fidFloatingPvAmort(AnchorDate As Date, Start As Variant, Maturity As Variant, Tenor As String, DayCountBasis As String, DayRule As String, Notional As Double, FwdCurveMat As Variant, FwdCurveRates As Variant, Method As String, Optional DiscCurveMat As Variant,Optional DiscCurveRates As Variant) As Double

Dim temp As Variant, SDate As Date, EDate As Date, Cvg As Double
Dim i As Integer, n As Integer, noti As Double

If IsMissing(DiscCurveMat) Or IsMissing(DiscCurveRates) Then
   DiscCurveMat = FwdCurveMat
   DiscCurveRates = FwdCurveRates
End If

temp = fidGenerateSchedule(AnchorDate, Start, Maturity, Tenor, DayCountBasis, DayRule)

n = UBound(temp)
For i = 1 To n
SDate = temp(i, 3)
EDate = temp(i, 4)
```

```
Cvg = temp(i, 5)
noti = (n - i + 1) / n * Notional
fidFloatingPvAmort = fidFloatingPvAmort + Cvg * fidForwardRate(AnchorDate, SDate, EDate, DayRule, DayCountBasis, FwdCurveMat, FwdCurveRates, Method) * fidDiscFactor(AnchorDate, EDate, DiscCurveMat, DiscCurveRates, Method) * noti
Next i

End Function

Public Function fidSwapPvAmort(AnchorDate As Date, Start As Variant, Maturity As Variant, FloatTenor As String, FloatDayCountBasis As String, FixedTenor As String, FixedDayCountBasis As String, DayRule As String, FixedRate As Double, TypeFlag As String, Notional As Double, FwdCurveMat As Variant, FwdCurveRates As Variant, Method As String, Optional DiscCurveMat As Variant, Optional DiscCurveRates As Variant) As Double

If IsMissing(DiscCurveMat) Or IsMissing(DiscCurveRates) Then
DiscCurveMat = FwdCurveMat
DiscCurveRates = FwdCurveRates
End If

Dim Fixed As Double, Floating As Double

Fixed = FixedRate * fidAnnuityPvAmort(AnchorDate, Start, Maturity, FixedTenor, FixedDayCountBasis, DayRule, Notional, DiscCurveMat, DiscCurveRates, Method)
Floating = fidFloatingPvAmort(AnchorDate, Start, Maturity, FloatTenor, FloatDayCountBasis, DayRule, Notional, FwdCurveMat, FwdCurveRates, Method, DiscCurveMat, DiscCurveRates)

If LCase(TypeFlag) = "receiver" Then
fidSwapPvAmort = Fixed - Floating
Else
fidSwapPvAmort = Floating - Fixed
End If

End Function
```

1.5 Pricing and risk managing amortizing swaps

1.5.1

The present value of the amortizing swap is 34,061,494. We accept a certain amount of slack here (plus or minus 1 million), since slight calibration differences will result in a different PV.

1.5.2

Using your newly developed pricing function yields.

Model rate delta vector			
	SwapAmort		

Knot points	Disc	Fwd	Net
22-apr-16	-952	-2,211	-3,163
24-apr-17	-1,178	-3,809	-4,987
23-apr-18	-2,750	-14,349	-17,099
22-apr-20	-2,819	-24,422	-27,241
22-apr-22	-2,106	-50,693	-52,799
22-apr-25	-1,040	-33,683	-34,724
23-apr-35	9	338	346
24-apr-45	0	0	0
Total	-10,836	-128,829	-139,665

Market rate delta vector			
		SwapAmort	
Start	**Maturity**	**IRS**	**CCS**
2b	1Y	-3,097	-721
2b	2Y	-4,708	-945
2B	3Y	-15,884	-2,598
2B	5Y	-25,676	-3,506
2B	7Y	-53,246	-3,592
2B	10Y	-37,876	-1,679
2B	20Y	409	31
2B	30Y	5	-1
	Total	-140,076	-13,011

Again, we accept slight differences in the risk numbers.

1.5.3

Making use of the FID functions (not your own) yields.

Model rate delta vector			
	Swap		
Knot points	**Disc**	**Fwd**	**Net**
22-apr-16	-950	210	-741
24-apr-17	-1,218	55	-1,163
23-apr-18	-3,472	-1,441	-4,912
22-apr-20	-4,133	-4,142	-8,275
22-apr-22	-3,759	-14,433	-18,192
22-apr-25	-4,012	-218,642	-222,654
23-apr-35	9	106	115
24-apr-45	0	0	0
Total	-17,535	-238,286	-255,822

Market rate delta vector			
		Swap	
Start	**Maturity**	**IRS**	**CCS**
2b	1Y	-463	-469
2b	2Y	-596	-604
2B	3Y	-2,334	-2,370
2B	5Y	-3,899	-3,967
2B	7Y	-8,863	-9,070
2B	10Y	-242,322	-8,103
2B	20Y	65	67
2B	30Y	1	1
	Total	-258,413	-24,515

Again, we accept slight differences in the risk numbers.

1.5.4

We immediately realize that the outright risk for the amortizing swap is smaller than that of the regular swap. This is obvious, since we have a smaller notional (especially towards the end).

The disc risk starts out almost identical, since the 4% rate (which is deep in-the-money for us a receiver) is used on the same notional. That means the cash flows will be similar. Further out the curve we still have negative disc risk (remember, we are in-the-money), but of a much smaller magnitude, since the notional has amortized.

The forward curve risk is where things really look odd. This is a tough question! Remember that if we bump a *spot* point on the zero coupon forward/Libor curve, say 5Y, we have to keep all other points on the zero coupon curve unchanged. Think marginal versus averages. This means that the forward rates (marginal rates) up to 4Y are unchanged, *higher* between 4Y and 5Y, and *lower* between 5Y and 6Y. For the standard swap we only have risk at maturity, since bumping any other point will have an offsetting effect the next year. However, for the amortizing swap these effects no longer net out, since a later date generally has a lower notional. Hence, bumping the 5Y point will yield negative PV (we are receiving) in the 5Y bucket, and a smaller positive PV in the 6Y bucket. Thus the negative risk in buckets before maturity.

2. Swaptions

2.1

SABR models the stochastic nature of volatility and has dynamics that better reflects the actual behavior of interest rates. For example, the vol level usually decreases when rates rise and the delta must be able to account for this effect – we have to add a “correction term” to the Black delta given by Black vega x Vol delta (see slides).

The answer is not that SABR can fit the skew observed in the market – the Black model can do this if we remember to use different vols for different strikes.

2.2

We are short a payer, so should pay fixed in the hedge.

2.3

The ATM swaption has a delta close to 50%, so the notional should be around €50mm. The hedge notional will have to be adjusted dynamically.

2.4

The delta of the payer decreases, so you should decrease the notional in the hedge.

2.5

You are short vega, so you need to buy straddles.

2.6

This is a "trick" question: the straddles you have bought is a perfect vega hedge, so you should neither buy nor sell.

Partial credit is given to those who correctly reason that vega in the client position decreases and therefore you would sell back straddles.

2.7

A swaption is an option on a portfolio of future Libor fixings whereas a cap is a portfolio of options on future Libor fixings. Since $\max(\sum x_i, 0) \leq \sum \max(x_i, 0)$, the cap is always worth at least as much as the swaption, so in that sense it's a good hedge. Whether the cap is a cheap hedge or not is beside the question.

Of course, buying the cap will also hedge most of your delta, so strictly speaking you need to unwind your delta hedge from before – it's fine if you mention this in your answer.

2.8

We expect the curve to move down in a more or less parallel fashion, so we gain on the short payer (negative DV01).

2.9

We expect the curve to flatten as the 20Y rate moves up and the 30Y rate move down.

2.10

A short payer has positive sensitivity to the 20Y point (want 20Y rates up) vs a negative sensitivity in the 30Y point (want 30Y rates down), so we gain on both accounts.

2.11

It's a gamma trade (short-dated options). A straddle has close to zero delta, so should not worry about delta at inception.

2.12

No, you have already received the premium and the swaption is cash-settled so will always be a liability to you – hence you have no credit risk. If the swaption was physically settled, there would be some credit risk as the underlying swap might move in our favor after the client had exercised the swaption.

2.13

The ASW spread is a measure of credit risk relative to the Cibor panel banks and the credit quality of the issuer is worse than the average bank in the Cibor panel.

2.14

A market maker (the bond desk in this case) always trades at the side which is most favorable to him/her. The bond desk is paying the spread, so you are transacting at his/her bid, i.e. you receive Cibor6m + 295 bps.

3. The SABR model

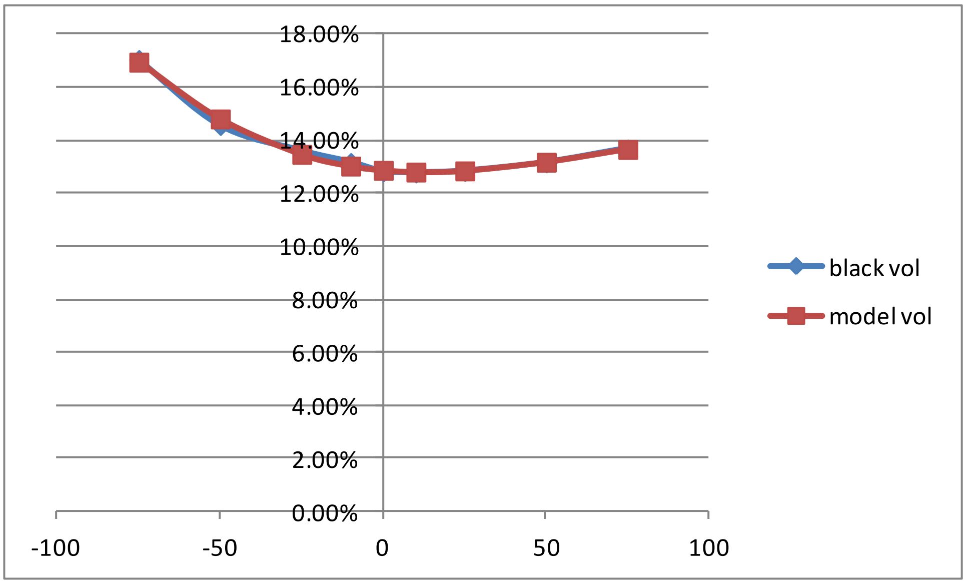
3.1.1

offset	black vol
-75	17.05%
-50	14.56%
-25	13.59%
-10	13.17%
0	12.82%
10	12.77%
25	12.83%
50	13.15%
75	13.68%

3.1.2

offset	black vol	model vol	diff
-75	17.05%	16.95%	0.10%
-50	14.56%	14.81%	-0.25%
-25	13.59%	13.47%	0.12%
-10	13.17%	13.03%	0.14%
0	12.82%	12.87%	-0.05%
10	12.77%	12.80%	-0.04%
25	12.83%	12.85%	-0.02%
50	13.15%	13.18%	-0.03%
75	13.68%	13.66%	0.02%

3.1.3



alpha	80.00%
sigma_0	5.06%
epsilon	25.98%
rho	-3.10%

3.2.1

You implicitly assume that your ASW spread will remain unchanged. If your credit quality deteriorates relative to the Euribor panel banks (ASW spread up), then your all-in rate could be higher than 2.5%.

You do not assume that your credit quality is worse than the Euribor panel banks – that's a *fact*, not an assumption.

3.2.2

Receiver swaption should be struck 1.0683% for collar to have zero PV.

4. Digital options

4.1.1

We distinguish two possibilities:

- For ATM digitals net vega is close to zero and the price depends mostly on the skew
- For ITM (OTM) digitals net vega is negative (positive) so level as well as skew is important

It's acceptable to focus on the first case and state that slope is the most important feature as the call spread replication is what you should have in mind.

4.1.2

The PV of the digital caplet divided by the discount factor times the coverage factor equals the probability of $L \geq K$ under the $T+\delta$ forward measure, where T is the fixing time of the underlying Libor rate and δ is the coverage factor. The numeraire is the zero-coupon bond maturing at time $T+\delta$.

4.1.3

The implied probability of $L \geq F$ in the Black model will be slightly below 50%. This is because the lognormal distribution is right-skewed, so the mean is greater than the median. By definition, the implied probability of fixing above the median is 50%, so the implied probability of fixing above the mean – which is equal to F – is less than 50%.

4.1.4

The implied probability of $L \geq F$ in the normal model will be exactly 50%. This is because the normal distribution is symmetric, so the mean is equal to the median. By definition, the implied probability of fixing above the median is 50%, so the implied probability of fixing above the mean – which is equal to F – must also be 50%.

4.2.1

The note can be decomposed as follows:

- A fixed annuity paying 3% semi-annually
- A short position in a spot-starting digital cap (= 9 caplets) struck at 3.75% paying 3%
- A short position in a spot-starting digital floor (= 9 floorlets) struck at 0.50% paying 3%

4.2.2

Note: The SRN was set up to have above-par PV which is a bit unrealistic, i.e. if ACME Bank were to issue this note at par it would make a loss upfront. Many of you got this right and also commented on the unrealistic aspect.

annuity	1,446,857
cap	22,818
floor	30,507
notional	9,211,965
total pv	10,605,498

4.2.3

upfront fee	-605,498